EEG FREQUENCY ANALYSIS ON THE PDP LAB 8/E COMPUTER SYSTEM

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

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PDP LAB 8/E COMPUTER SYSTEM

by

Lawrence Morrison Gorham

September 1974

Thesis Advisor: G. K. Poock

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EEG Frequency Analysis on the

PDP Lab 8/E Computer System

by

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ABSTRACT

This thesis describes the analysis, method and computer programs used to obtain the Fast Fourier Transformation (FFT) of an electroencephalogram (EEG) using a small laboratory computer like the PDP Lab 8/E. The EEG power spectrum was then computed from this transformation. The information contained in this thesis is intended to enable the user to compute the Fourier coefficients of a set of data points or compute the power spectrum of a real waveform such as the EEG.

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I. INTRODUCTION

As cerebral cortex cells discharge electrically, a small potential is produced; when this potential is averaged with the other discharging cells it can be sampled with electrodes applied to the scalp. This weak voltage is amplified 100,000 times by a low noise, high gain amplifier. For simplicity and economy, a paper and pen writing oscillograph is used to record the changing potential. This record of cortical potentials is called an electroencephalograph (EEG).

These waves are very complicated and analyzing them has been a problem since their discovery. Due to the wave-like nature, EEG frequency or spectrum analysis has been the subject of much research over the years. Large, high speed computers are being used to aid this work today. The purpose of this study was to develop fast Fourier techniques for spectrum analysis of EEGs using a small laboratory computer like the PDP Lab 8/E computer system. With the computer located in the laboratory, near subject and equipment, it was hoped that some kind of real time frequency analysis could be accomplished.

II. BACKGROUND

A. PHYSIOLOGY

The small potential measured in the EEG originates in the outer .1 inch of the brain called the grey matter. The working cells of the grey matter are the neurons, numbering perhaps to ten billion in all. Very little is known about how these cells operate to provide the many functions that the brain accomplishes. One well known and supported theory by John C. Eccles explains how the EEG waveforms are produced [Ref. 10]. Each neuron is assumed to be an independent functioning unit, receiving stimuli from outside sensory devices and other neurons through dendrite fibers. When a particular number of stimuli are received, the neuron fires a pulse of electricity to other neurons via axon fibers. Receiving reverse polarity signals will cancel the effect of incoming stimuli and prevent the neuron from firing. In this way each neuron may be connected to as many as 100 other neurons.

Usually many thousands of neurons are stimulated simultaneously. As they reach their excitation points and fire, other thousands of neurons receive these new stimuli. Thus the collection of stimuli moves through the brain step by step. Each stage in the path may be only a thousandth of an inch or it may be three or four inches in another part of the brain. After firing, each neuron has a deactivated period of several seconds, in which it cannot be stimulated

to respond. This delay, or refractory period as it is called, produces a wave-like quality to the stimuli collection as it moves along. Eccles believes that this explains the wave-like nature of cortical potentials as they filter out through the skull and scalp and are recorded on the EEG.

Another important aspect of these neural pathways is that they frequently double back on each other. This causes a circular path to form, which provides a self-exciting feature to the brain. These closed loops, with fixed distance and refractory periods, may account for the predominance of certain frequency waves. The prospect of neurons having feedback loops opens up a whole new level of complexity to the brain. In this way each neuron could be viewed as a very flexible amplifier, each one operating differently, depending on the kind of feed-back it is receiving.

B. HISTORY

A short history of the cortical potential must begin with Richard Canton, a British physiologist, who reported in 1875, "Feeble currents of varying direction pass through the multiplier when electrodes are placed on two points of the external surface (of the brain), or one electrode on the grey matter, and one on the surface of the skull." At the time, Canton was examining cat, monkey and rabbit brains with a sensitive galvanometer using optical magnification. It is remarkable that he could distinguish such small potentials without the use of electronic amplification, which

wasn't developed for another fifty years. In 1876 the Russian, Danilevskey, discovered a change in the average potential of the cortex in response to acoustic stimuli. Only recently has this interesting aspect of brain activity been reinvestigated. In 1914 Cybulski recorded an apileptic seizure in a dog. The early attempts at recording cortical potential used photography, however, this was expensive and tedious. Pen recorders were not available for this use until 1940 [Ref. 8].

Recording cortical potentials in man lagged behind their demonstration in animals for many years. In man, both electrodes were attached to the scalp, causing the potentials to be greatly attenuated through the skull and scalp. The first recordings from the human brain were made in 1924 by Hans Berger in Germany. He worked and published much, exploring many aspects of the use of the electroencephalogram. Some of these included physiological, neurological, psychiatric and psychological applications. He confirmed Canton's findings in animals and established that these waveforms change with age, with sensory stimuli and with chemical changes in the body. He was the first to record a major epileptic seizure in man. Due to the earlier work by Kaufman and Cybulski, it became known that epilepsy in animals caused abnormal waveforms.

After Berger's first publication in 1929, many laboratories took up this work and knowledge in the field grew rapidly. After substantiating Berger's claims, the various

brain centers began to specialize in different research areas: Gibbs and Lennox in epilepsy, the Davises in normal subjects, Adrian, Hoagland and Jasper in the physiological mechanisms and Walter in brain lesions. The growth of electronencephalography has really paralleled the development of the machinery for recording the waves. In the 1930's the electronic amplifier gave a tremendous boost to the field. In 1948 the transistor allowed the development of highly portable equipment. Modern instruments allow 16 channels to be monitored simultaneously, each of which has linear response and accurately reproduces frequencies from 1 to 500 Hz. A more complete history of electroencephalography may be found in Braizier [Ref. 4].

One of the many things that Hans Berger discovered about the human electroencephalograph was that the most normal pattern in adults, with eyes closed, was a frequency between 8 and 13 Hz which he called alpha rhythm. These and other rhythms seem to originate in the cortex, however, they cease if the cortex is surgically disconnected from the thalamus. Alpha rhythm is most strong when the subject is relaxed with eyes closed. Opening the eyes in normal light significantly reduces alpha rhythm and similarly, opening the eyes in total darkness causes a temporary reduction. There is some evidence [Ref. 18] to support a positive correlation between high amplitude alpha rhythms and a passive or introverted personality structure as revealed by psychoanalysis. Similarly, low amplitude alpha rhythms were found prevalent in extroverts [Ref. 18].

Delta activity is between 1 and 3.5 Hz and is generally associated with sleep. In 1954 Bremer achieved a strong delta state by disconnecting all the sensory pathways. This is logical since sleep is most easily induced when sensory stimulation is reduced to a minimum. It must be emphasized however, that even though no light, sound, etc. is sensed, one does not sleep unless he is "tired," i.e., delta state and sleep are not the same. Theta activity, in the 4 to 7 Hz range, is usually associated with normal children [Ref. 4]. The percentage of theta decreases significantly with age in normal subjects. Beta activity, with a frequency of 14 to 30 Hz, is generally associated with increased anxiety levels. The incidence of beta is increased in subjects under-going alcohol withdrawal. Beta activity can easily be blocked in the motor center associated with a particular movement[Ref. 4].

The four basic waves, alpha, delta, theta and beta, span the frequency spectrum of cortical potentials. Another pattern of interest is the mu rhythm, which is characterized by a sharp peak in negative half-cycle, rounded positive half-cycle and 7 to 11 Hz. Mu rhythm is reduced not only by overt movement of a limb but also in some cases when the subject is contemplating movement [Ref. 4, 8].

Except for beta waves, all the waveforms previously described tend to be reduced by some kind of stimulation. In contrast, lambda waves, a saw-toothed waveform, are best seen when the subject is actively looking at something of

interest to him. This waveform usually occurs when the subject is scanning a complex pattern or picture. Paradoxically, random waves of the same polarity and shape occur in some subjects during light sleep. A thorough analysis of the various waveforms in psychologically normal and abnormal subjects may be seen in Thorp and Heyman [Ref. 24] and Kiloh and Osselton [Ref. 21].

Interpretation of the EEG by visual inspection of the record depends in a large part of recognizing common patterns and their association with psychologically normal or pathological conditions. Unfortunately these visual methods are unable to differentiate between clinical conditions having similar EEG's. Several different methods for analyzing the EEG are being tested. One of the first attempts was to get a quantitative grasp on the amplitude of the waves. Walter [Ref. 4] joined the peaks and troughs with two lines and then measured the average distance between the lines. With the advent of amplifiers, electronic integration solved this problem.

In looking for better ways to specify cortical potentials, many forms of frequency analysis have been tried. As early as 1932, Dietsch suggested that these potentials might be composed of sine waves and he outlined the procedure whereby Fourier analysis could be used to determine the frequency components present [Ref. 9].

The first frequency analysis of an EEG was accomplished at the Cincinnati College of Medicine in 1944, where the frequencies were determined by manually counting the number of

complete waves in each second [Ref. 12]. In the late forties and fifties, much work was stimulated by Gibbs and Grass [Ref. 13] when wide-band electronic filters were used to separate the different frequencies [Ref. 19, 22]. Filter devices are commonly used in this way, even today [Ref. 25]. An excellent review of all work on EEG analysis techniques up through the fifties may be found in Reference 5.

Spectrum analysis of physiological data using the discrete Fourier transform was accomplished by Halberg in 1961 [Ref. 14, 23]. Walter was first to use the discrete transform on cortical potentials in 1963 [Ref. 26, 28]. Most of the quantitative frequency work being done on EEG's today is with the aid of large computers in which the discrete Fourier transform provides the conversion [Ref. 11, 15, 17]. The U. S. Navy is also entering this research area, at the Navy Medical Neuropsychiatric Research Unit in San Diego, California [Ref. 15]. In 1973 the Fast Fourier Transform, a faster version of the discrete Fourier transform, came into use on the EEG [Ref. 2].

III. FOURIER ANALYSIS

A. FOURIER TRANSFORM

1. Continuous

An introduction to Fourier analysis must begin with the general form of the Fourier series. It is

$$f(t) = A_o + \sum_{n=1}^{\infty} A_n \cos n\omega t + \sum_{n=1}^{\infty} B_n \sin n\omega t \qquad (1)$$

where $\omega = 2\pi/T$ and T is the period. Any waveform can be represented by determining the coefficients A_o, A_n and B_n and summing all the terms of this series. The coefficients give the relative contribution of each frequency that is present in the signal. These are harmonics or integer multiples of the fundamental frequency 1/T. Fourier developed a set of equations from (1) and solved them simultaneously. The result

$$A_o = \frac{1}{T} \int_o^T f(t)dt \qquad (2)$$

$$A_n = \frac{2}{T} \int_o^T f(t) \cos n\omega t dt \qquad (3)$$

$$B_n = \frac{2}{T} \int_o^T f(t) \sin n\omega t dt \qquad (4)$$

provides a method for finding the harmonics when the function is known.

2. Discrete

Frequently, however, the complete function is not known and only a discrete number N of samples over a finite

period T is known. In this situation, an approximation to the continuous function may be written

$$f(t) = A_o + 2\sum_{m=1}^{n} A_m \cos m\omega t + 2\sum_{m=1}^{n} B_n \sin m\omega t \quad (5)$$

where n = N/2 for N even and n = N+1/2 for N odd. The expressions for the coefficients now become

$$A_m = \frac{1}{N}\sum_{r=-n}^{n-1} f(t) \cos \left(\frac{2\pi mt}{N} \right) \quad (6)$$

$$B_m = \frac{1}{N}\sum_{r=-n}^{n-1} f(t) \sin \left(\frac{2\pi mt}{N} \right) . \quad (7)$$

These functions are easily computed from the set of sample data points; however, the time required is sometimes prohibitive. It will be noted that this procedure requires N^2 trigonometric operations, since each of N coefficients is computed from N terms of a series. These computations, in addition to numerous other arithmetic operations, require considerable computer time, especially on small laboratory computers like the PDP Lab 8/E.

B. PROBLEM AREAS

There are three problems commonly encountered when using the discrete Fourier transform. These are: aliasing, leakage and the picket-fence effect. Aliasing occurs when a higher than expected frequency is sampled, but appears in the spectrum as a lower frequency. Suppose an unknown signal is being sampled five times at a rate of 100 Hz. If an 80 Hz component is present, it will appear in the spectrum

as 20 Hz and there is no way of distinguishing this pseudo-frequency from the real one. Nyquist proposed the solution, in that, the sampling rate should be twice the highest frequency present [Ref. 16]. This can be accomplished in an unknown signal by passing it through a lowpass filter set at half the sampling rate. A lowpass filter was used to prevent aliasing in this study.

The problem of leakage occurs in all Fourier analysis using finite sampling records. The record, or the total length of the sample, can be viewed as a window. When this window is transformed, without the sample data present, the resulting spectrum contains not only the fundamental frequency but also spurious peaks called sidelobes. When the window with its sample is transformed, these sidelobes tend to leak into the spectrum. Because of the relatively small sidelobes, in the frequency range of interest, leakage was not a problem in this study.

The picket-fence effect is a problem when a pure sinusoid occurs between two adjacent harmonics. If it is exactly halfway between computed harmonics, the signal is reduced to .637 of the original in both harmonics. The worst situation occurs when there is a frequency between each pair of adjacent harmonics. In this case an erroneous ripple, or picket-fence, appears in the spectrum. Pure sinusoids were assumed not to be present in EEG's and thus the picket-fence effect was ignored. Bergland discusses these problems as well as the mathematics associated with them in Reference 3.

IV. FAST FOURIER ANALYSIS

A. FAST FOURIER TRANSFORM

In 1965 James Cooley and John Tukey published an important paper that described a much faster method to compute the discrete Fourier transformation, called the Fast Fourier Transform (FFT) [Ref. 7]. The procedure adopted for use on high speed computers has become known as the Cooley-Tukey Algorithm. A general outline of the proof of this algorithm follows.

A time series X_k containing N even samples, can be partitioned into two functions Y_r and Z_r, each containing N/2 points. Let Y_r be composed of the even-numbered points $(x_o, x_2, \ldots, x_{N/2})$ and Z_r the odd-numbered points $(x_1, x_3 \ldots x_{N-1/2})$. Equations (6) and (7) may be combined using complex number notation, so that

$$A_m = \sum_{r=0}^{n-1} X_r \; W^{mr} \qquad (8)$$

where $W = e^{2\pi j/N}$ and $j = \sqrt{-1}$. Since Y_r and Z_r are a set of N/2 points, each has a discrete Fourier transform whose coefficients are

$$B_m = \sum_{r=0}^{N/2-1} Y_r \; W^{mr} \qquad (9)$$

$$C_m = \sum_{r=0}^{N/2-1} Z_r \; W^{m(2r+1)} \qquad (10)$$

with $m = 0, 1, \ldots, \frac{N}{2}-1$. Combining equations (9) and (10), to get odd and even points, gives

$$A_m = \sum_{r=0}^{N/2-1} [Y_r W^{mr} + Z_r W^{m(2r+1)}]$$

$$A_m = \sum_{R=0}^{N/2-1} Y_r W^{mr} + W^m \sum_{R=0}^{N/2-1} Z_r W^{mr}$$

$$A_m = B_m + W^m C_m \qquad (11)$$

$$A_m + N/2 = B_m - W^m C_m . \qquad (12)$$

The details of this proof will be found in Reference 6.

If X_r is an even power of two, then Y_r and Z_r also have an even power of two points and each can be partitioned again and again until only one point remains in each set. The Fourier transform of a single point is just that point again; the entire transformation can be accomplished using equations (11) and (12) repeatedly at each partition. The requirement that the number of points N be a power of two assures that after $\log_2 N$ partitions there will be only one point in each set. In this way, the entire set of points can be translated without carrying out the manipulations of equations (6) and (7). Since the FFT is derived from the discrete Fourier transform, the FFT maintains the same precision as before.

B. FLOWPATH

An example of the FFT for N = 8 will be illustrative. Figure 1 shows the flow diagram and the configuration after the first partition. Using Tukey's notation, each dot represents a variable and the factors entering the dot are additive. When a multiplication is required before adding,

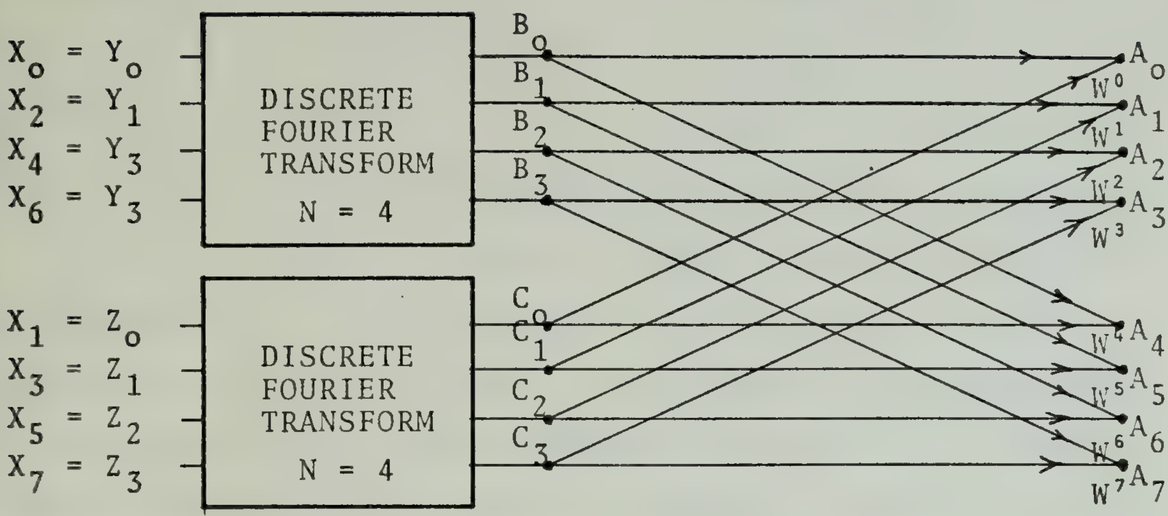

Figure 1. Fast Fourier Transform Flow Graph, First Partition.

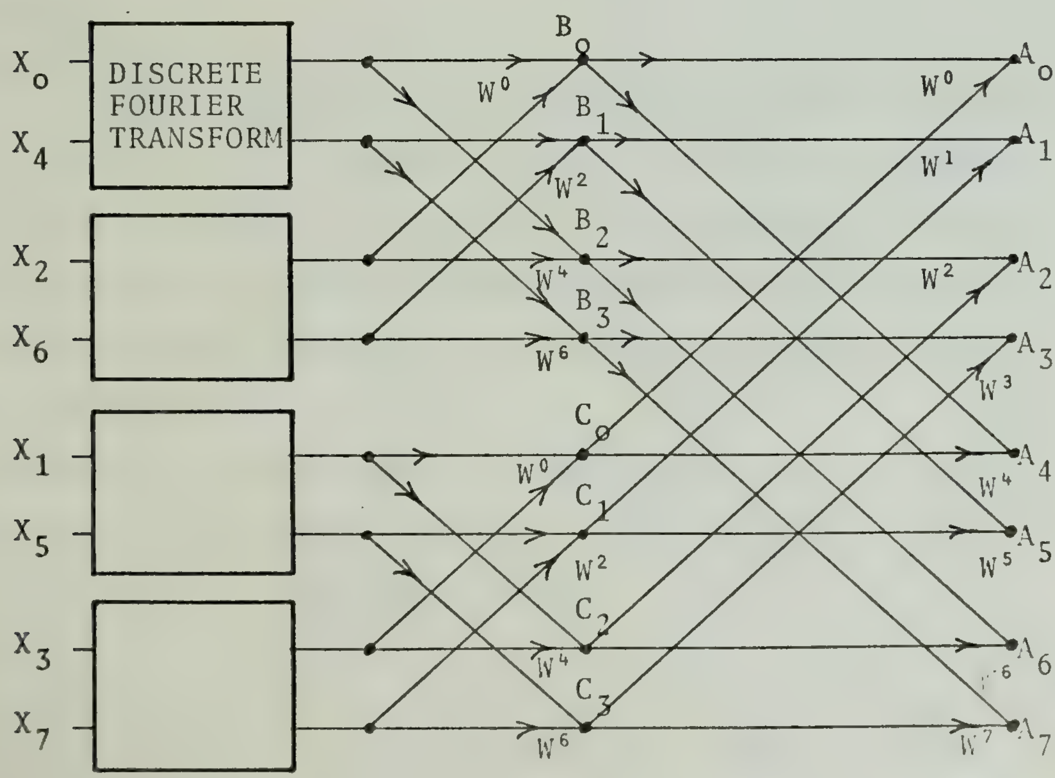

Figure 2. Fast Fourier Transform, Second Partition, Flow Graph.

the quantity will be beside the factor to be multiplied. The set of A_j for $j = 0,\ldots,7$ is computed from equations (11) and (12). The quantity near the bottom right dot is $A_7 = B_3 + W^7C_3$. Figure 2 shows all quantities after two partitions and Figure 3 shows the complete transformation broken down into only multiplication and addition operations.

It may be noted in Figure 3 that the data points must be rearranged according to the bit reversal rule before the algorithm can be carried out. Bit reversal is accomplished by representing the data point position number in binary and reversing the bits about the center. Position (3) is written (0 1 1) and is reversed to (1 1 0), which is (6). Thus (0) stays a (0), (1) becomes a (4) and so on. More information about FFT may be found in References 3 and 6.

C. PROGRAM

The computer program FAST was written to accomplish the FFT on the PDP Lab 8/E computer system. A complete listing of FAST may be found in Appendix A. The program is written in BASIC, one of the commonly used languages on the PDP Lab 8/E. FAST receives data points as inputs and outputs the Fourier coefficients.

In order to operate the FFT, read FAST into core using normal methods, and then type RUN. The program will type

```
ENTER DATA POINTS
?
```

and go into a wait loop. Type in 64 data points in their proper order. The number of data points may be changed to

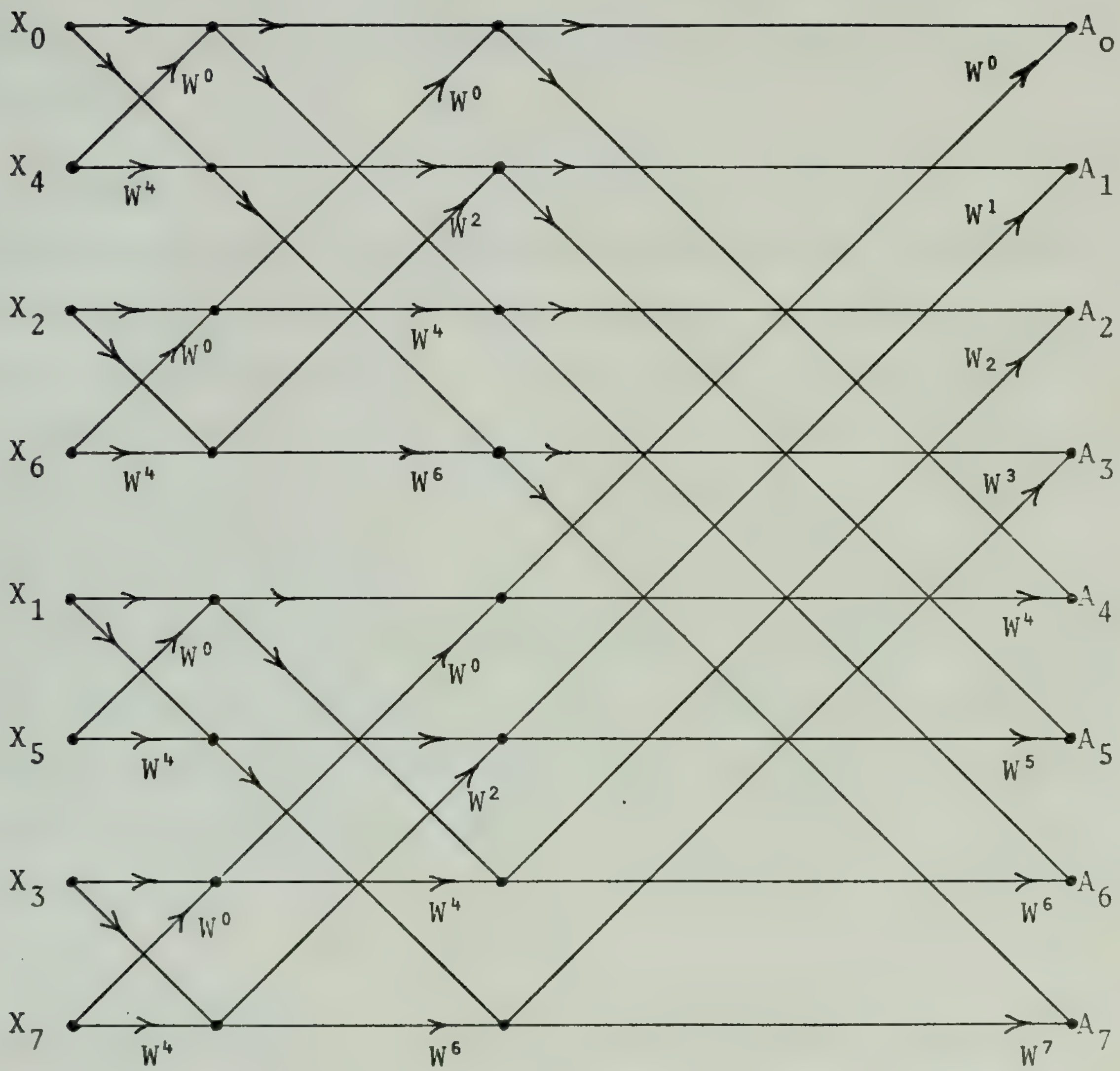

Figure 3. Fast Fourier Transform, Flow Graph, Last Partition.

any power of two, but cannot exceed 64 due to core limitations, which is 8,000 words in the PDP Lab 8/E. FAST will translate the data points and type the Fourier coefficients. The time required to convert 64 points is approximately 25 seconds. The power associated with each harmonic can be obtained by squaring and adding the coefficients at each frequency.

D. EXAMPLE

As an example to illustrate the use of FAST, the square wave shown in Figure 4 was translated and the results compared with the coefficients from the continuous Fourier transform. Table I shows the result of a FAST translation of the square wave along with the coefficients. To determine the A_n coefficients using the continuous transform, equations (2) and (3) are integrated from -T/2 to T/2. Since f(t) is an odd function, the A_n terms are all zero. Using equation (4), however, and integrating over the same range, it is easily shown that

$$B_n = \frac{2 - 2\mathrm{Cos}\,n\pi}{n\pi} \quad . \tag{13}$$

To compare with FAST, the same 32 harmonics were computed using equation (13)(see Table II). The answers from FAST are within two decimal places of the answers computed from the continuous Fourier transform.

Figure 4. Square Wave.

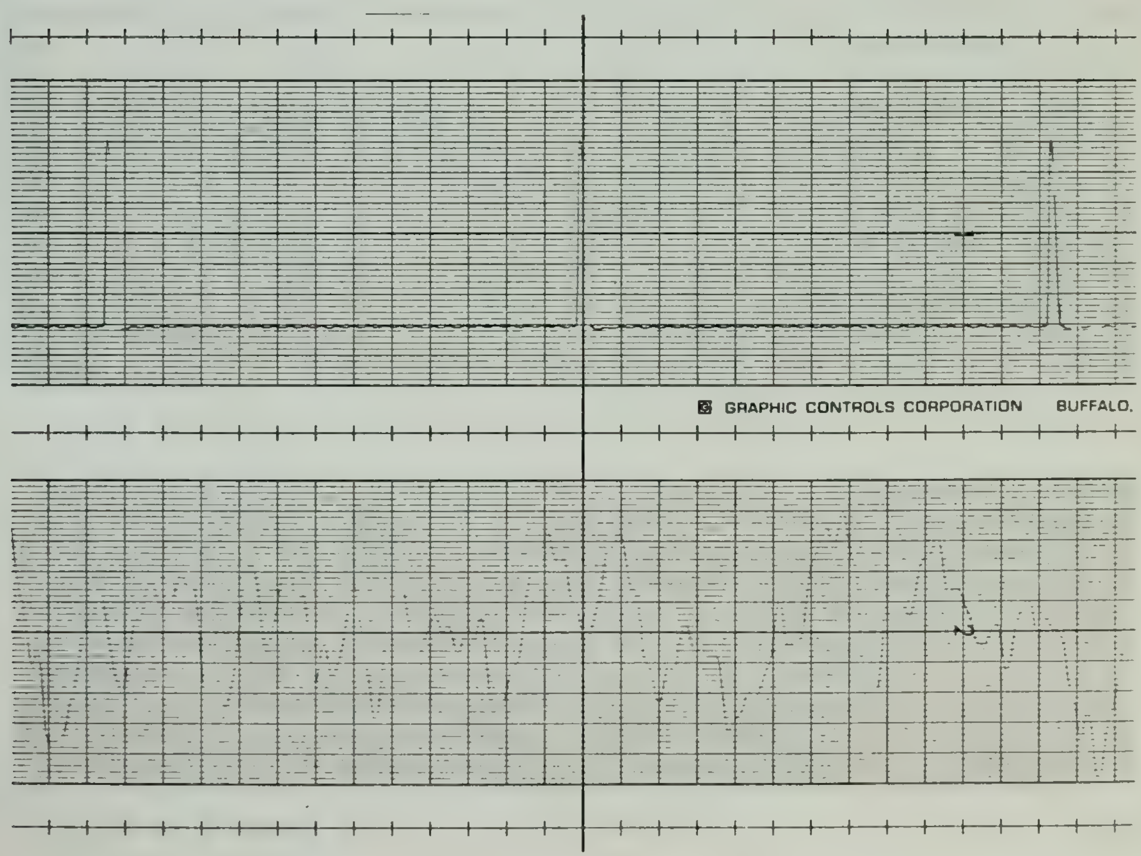

Figure 5. EEG Wave and Timing Pulse.

HARMONIC	A(N)	B(N)
0	0	0
1	0	1.272217
2	0	0
3	0	.4213407
4	0	0
5	0	.2495139
6	0	0
7	0	.1746758
8	0	0
9	0	.1321451
10	0	0
11	0	.1042749
12	0	0
13	0	.08427145
14	0	0
15	0	.06895806
16	0	0
17	0	.05664668
18	0	0
19	0	.04635313
20	0	0
21	0	.03746103
22	0	0
23	0	.02956024
24	0	0
25	0	.02236283
26	0	0
27	0	.01565539
28	0	0
29	0	9.270938E-03
30	0	0
31	0	3.070265E-03

READY.

Table I. FAST Coefficients Translated from the Square Wave of Figure 4.

RUN

HARMONIC	B(N)
0	-8.507059E+37
1	1.273241
2	-4.743191E-09
3	.4244135
4	-2.371595E-09
5	.2546481
6	-1.581064E-09
7	.1818915
8	-1.185798E-09
9	.1414712
10	-9.486382E-10
11	.1157491
12	-7.905318E-10
13	.09794159
14	-6.775987E-10
15	.08488271
16	-5.928989E-10
17	.07489651
18	-5.270212E-10
19	.06701267
20	-4.743191E-10
21	.06063051
22	-4.311992E-10
23	.05535829
24	-3.952659E-10
25	.05092963
26	-3.648608E-10
27	.04715706
28	-3.387993E-10
29	.04390485
30	-3.162127E-10
31	.04107228

READY.

Table II. Continuous Fourier Transform of B_n Coefficients for Square Wave of Figure 4.

V. SPECTRUM ANALYSIS

A. SPECTRUM

In order to do frequency analysis of EEG waveforms, a computer program called SPECTRUM was written which incorporated FAST (Appendix A). The problems encountered were many. First, a relatively noise-free signal had to be presented to the PDP Lab 8/E analog-to-digital (A-D) converter. Then the signal had to be sampled at the right rate and duration. Finally, the data had to be delivered to FAST in a compatible format for translation. The power spectrum was chosen as the output format since it has more intuitive appeal than some other measure such as voltage or current. According to Parseval's theorem, the power at each frequency can be computed by squaring the two coefficients and adding them together. This was done and the power spectrum is displayed on the teletype as percent of total power at each harmonic.

B. APPARATUS

The Beckman type RM Dynograph Recorder was used to pickup and amplify the cortical potentials. Using sodium chloride paste, silver electrodes were applied to the subject's scalp at positions C_4 and T_6. A standard Beckman A-C coupler was used with the TIME CONSTANT switch set to .03 and HIGH FREQ switch on 3. Depending on the signal strength, the preamplifier was set to either 1 mv/cm or .5 mv/cm and

the power amplifier set for a multiplication of .02. This adjustment was made in order to maintain a relatively constant signal level from the power amplifier. The FILTER switch was set on 3, which is a 60 Hz lowpass filter.

The EEG was recorded on one track of a Hewlett-Packard, four track, instrumentation recorder on 1.0 mil magnetic tape at 3-3/4 in/s tape speed. A seven ms timing pulse was recorded on another track from an Ortec pulse generator. The timing pulse made it possible for the computer to find a particular place on the tape. The two tracks were then connected to the A-D converter of the PDP Lab 8/E computer. As an alternative hookup, the A-D converter can be connected directly to the Beckman and sample the EEG as the subject produces it.

C. SAMPLING RATE

With the proper signal on channel 1 of the A-D converter, SPECTRUM samples and translates it to the power spectrum. According to Nyquist's rule, the sampling rate must be twice the maximum frequency. The maximum frequency passed by the Beckman filter is 60 Hz, so the sampling rate must be at least 120 Hz. In order to give a little extra margin, 128 Hz was chosen so that the harmonics would come out in even multiples of two. FAST can only convert 64 points and sampling at 128 Hz, the period (T) of the sample is ½ second. The reciprocal of the period (1/T) determines the increment of the harmonic. In this case 1/T = 2, thus each harmonic is exactly 2 Hz apart.

D. EXAMPLE

After loading SPECTRUM and typing RUN, the program samples, converts the data, and types the power spectrum. Table III shows an example of one SPECTRUM run, taken from a subject who was relaxing with his eyes closed. The program starts sampling when it receives a numbered timing pulse on channel three. The EEG waveform from which this example was taken, along with its timing pulse, is shown in Figure 5. SPECTRUM uses the timing pulses to locate the desired sample on the tape.

E. EXPERIMENT

By choosing the proper segment with the timing marks and replaying the tape 20 times, SPECTRUM was used to scan a length of tape arbitrarily set for 10 seconds and produce the power spectrum for each half second record. The results of this experiment were reduced to the graph of Table IV. It plots frequency against time, with each number representing the percentage of total power for a given frequency and time. If the table is viewed as a three dimensional graph, where percentage power is shown coming out of the page, it is easy to visualize a ridge running up the 10 Hz line. The subject was again resting with eyes closed, so the preponderance of alpha frequency (8, 10, 12 Hz) was understandable. The frequency also appeared to come in bursts. It was first on for about three seconds, then reduced for one second, then back on for a second, and so on for the length of the tape.

RUN

?

FREQUENCY HZ	POWER %
0	16.2738
1	1.157298
2	.07388068
3	4.684211
4	5.177504
5	3.729045
6	49.26551
7	5.911107
8	2.511872
9	3.593473
10	.7096686
11	.6507585
12	.9833005
13	.9566361
14	.6030759
15	.538035
16	.2399192
17	.2305483
18	.3325844
19	.2973473
20	.2739161
21	.1222699
22	.2039538
23	.178039
24	.1704812
25	.1763688
26	.1824439
27	.1651099
28	.1600951
29	.1526896
30	.1463384
31	.148733

READY.

Table III. A Typical SPECTRUM Run from a Half Second Sample of an EEG.

TIME (SEC) \ FREQUENCY (Hz)	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
10	1	9	4	30	12	19	8	3	1	3						1
	17			40		21	4	2		1	6					
9		12	18	2	17	21	6	5	4	1	4	5				
	3	5	10	13	8	35	2		7	2	4		4	2	1	
8		9	3	14	12	22	7	9	2	1			7	1	1	1
	8	21	9	8	35	1		1			3	2		1		
7	5	11	10	45		3	6	4	2	3	2	3				
		20	4	15	6	20	7	11	3			5	2			
6	7	8	16		1	38		9		4	1	5	2	1		
	13	10	10	13	3	20	4	4	6				2			1
5	6	21	30	4	3	6	1	4	4	1	8	1		1		
		29	14	9	18	6	5	6			5					
4	8	6	4	9	16	28	2	8	3		10					
	9		15	7	2	48		2			10					1
3	1	6	7	2	8	46	9		3	5	6	1	2			2
	3	24	3	2	7	42		2		3	8					
2	11	6	11	14	4	15	18	2	8	4	2		1	1		
	10	5	7	15	7	43	3							1	2	
1	19	7		11	2	39		4	1	4	5	2				
	9	16	12	17	8	3	6		8	4			1			

Table IV. Power Spectrum derived from an EEG Over a Period of Ten Seconds. Each Number Represents the Percentage Power at each Frequency for the Half Second Sampled. Zeros are omitted.

Sometimes this alpha burst effect can be detected right on the EEG.

There were two places where strong delta frequency was present. Prepresented on the graph by the 2 Hz line, delta was present in the beginning and again around the five second point. This normally indicates a very light stage of sleep, where the subject is drifting in and out of consciousness. Under questioning the subject indicated that he could have been asleep several times during the recording session. The graph also shows the rapidity with which frequency changes [Ref. 2, 11, 15 and 27]. One example of this is from the first half second to the one second point, where the 10 Hz power changed from 3% to 39%. Rather than changing smoothly, it was as if one frequency stopped and another started.

F. NOISE

Either because of a noisy amplifier or possibly a dirty tape, it may become necessary at some time to analyze a noisy signal. This will become known when repeated spectra from the same point show a variance. In order to reduce this variance, a method of averaging was devised. Instead of sampling at 128 Hz, a new rate of 1280 Hz was used and the sample means were computed every 10 points. The sample period was kept at ½ second, so a total of 64 sample means were computed each run. This reduced the variance by a factor of $\sqrt{10}$ and providing the signal was not too noisy, the results were satisfactory. Thus to obtain more information

from the noisy EEG, it may become necessary to sample more frequently than according to Nyquist's law.

Increasing the sampling rate is not without its shortcomings. Firstly, it takes longer to process the added data. Changing to a 10 point average increases the computing time from 25 to 40 seconds. Secondly, part of the data must be stored while the averages are being computed. The new program requires about 200 more storage positions, which is at the maximum capability of the PDP Lab 8/E with 8,000 word memory. Most spectral analysis work being done on EEG's today use a 200 Hz sampling rate [Ref. 2, 15].

VI. CONCLUSIONS AND RECOMMENDATIONS

A real time analyzer for the EEG was one of the goals of this study. This has only partly been realized since SPECTRUM requires 25 seconds to convert a half second sample. Since the EEG frequency pattern changes so rapidly, it is necessary to convert more than a half second; perhaps as much as 10 seconds in order to get a good representation of the changing nature of the wave. This means that a much larger computer is needed to do real time spectral analysis of EEG's.

As was shown using off-line techniques, SPECTRUM is a useful tool for spectral work. Using an instrumentation recorder in conjunction with some kind of pulsing device for timing, many different forms of frequency work may be accomplished such as autocorrelation, cross spectral analysis and digital filtering. SPECTRUM can also be used to compute power spectra from other kinds of continuous waveforms such as electrocardiograms and electromyograms.

The author believes that this area of study is an important one and should be extended in the future; however, there are limitations. The major limitation to this system resides in the lack of adequate data storage, both in the small core size, which is about 4,000 words with BASIC in core, and in the lack of a random access device such as a disk pack. Of these two, the disk pack would be most useful for spectrum analysis work since large amounts of data from

the A-D converter could be stored rapidly and processed as required. The flexibility of the entire system would be considerably enhanced because the disk pack could also be used to break up large programs which are now too big for the computer and then process them in a segmented fashion.

APPENDIX A

SPECTRUM

```
1    N=64
2    N1=63
5    DIM F(1,63),W(1,31),T(1)
8    SET RATE 6,558
10   DIM P1(300)
15   REAL TIME P1,1,0,900
18   IF ADC(3)<=.9GO TO 18
20   ACCEPT
30   FOR I=0 TO 63
35   FOR J=1 TO 14
40   F(0,1)=F(0,I)+ADB(0)
45   NEXT J
50   NEXT I
58   PRINT
100 FOR I=0 TO 31
110 READ F(1,I)
115 F(1,63-I)=63-F(1,I)
120 NEXT I
158 FOR I=0 TO N1
162 IF F(1,I)=I THEN 169
163 X=F(0,I)
164 J=F(1,I)
165 F(0,I)=F(0,J)
166 F(0,J)=X
167 F(1,I)=F(1,J)
168 F(1,J)=J
169 NEXT I
194 FOR I=0 TO N1
195 F(1,I)=0
196 NEXT I
210 W(0,0)=1
220 W(0,1)=COS(6.283185/N)
230 W(1,1)=-SIN(6.283185/N)
240 IF N<=4 THEN 295
250 FOR I=2 TO N/2-1
260 I1=I-1
270 W(0,I)=W(0,1)*W(0,I1)-W(1,1)*W(1,I1)
280 W(1,I)=W(1,1)*W(0,I1)+W(0,1)*W(1,I1)
290 NEXT I
310 J=1
320 FOR M=1 TO N
330 J=J*2
340 IF J>=N THEN 370
350 NEXT M
370 FOR K=1 TO M
380 J2=2↑(K-1)
390 I2=N/(J2*2)
```

```
400 FOR I=0 TO 12-1
401 T(0)=F(0,Y)
410 FOR J=0 TO J2-1
500 X=N*I/I2+J
510 Y=X+J2
520 Z=J*I2
530 T(0)=W(0,Z)*F(0,Y)-W(1,Z)*F(1,Y)
540 T(1)=W(1,Z)*F(0,Y)+W(0,Z)*F(1,Y)
550 F(0,Y)=F(0,X)-T(0)
560 F(1,Y)=F(1,X)-T(1)
570 F(0,X)=F(0,X)+T(0)
580 F(1,X)=F(1,X)+T(1)
590 NEXT J
600 NEXT I
610 NEXT K
690 A=0
700 FOR I=0 TO N/2-1
710 F(0,I)=(F(0,I)↑2)+(F(1,I)↑2)
712 A=A+F(0,I)
720 NEXT I
728 PRINT "FREQUENCY          POWER"
730 FOR I=0 TO N/2-1
732 A2=F(0,I)/A
750 PRINT I*2,A2
760 NEXT I
800 FOR I=0 TO N1
810 F(0,I)=0
820 NEXT I
900 DATA 0,32,16,48,8,40,24,56,4,36,20,52,12,44,28,60,2
901 DATA 34,18,50,10,42,26,58,6,38,22,54,14,46,30,62,1
998 RESTORE
999 GO TO 1
```

FAST

```
1    N=64
2    N1=63
5    DIM F(1,63),W(1,31),T(1)
30   FOR I=0 TO N1
35   INPUT F(0,I)
49   F(1,I)=0
50   NEXT I
58   PRINT
100  FOR I=0 TO 63
110  READ F(1,I)
120  NEXT I
130  FOR I=0 TO N1
140  I1=I*64/N
150  F(1,I)=F(1,I1)
155  NEXT I
158  FOR I=0 TO N1
162  IF F(1,I)=I THEN 169
```

```
163  X=F(0,I)
164  J=F(1,I)
165  F(0,I)=F(0,J)
166  F(0,J)=X
167  F(1,I)=F(1,J)
168  F(1,J)=J
169  NEXT I
194  FOR I=0 TO N1
195  F(1,I)=0
196  NEXT I
210  W(0,0)=1
220  W(0,1)=COS(6.283185/N)
230  W(1,I)=-SIN(6.283185/N)
240  IF N<=4 THEN 295
250  FOR I=2 TO N/2-1
260  I1=I-1
270  W(0,I)=W(0,1)*W(0,I1)-W(1,1)*W(1,I1)
280  W(1,I)=W(1,1)*W(0,I1)+W(0,1)*W(1,I1)
290  NEXT I
310  J=1
320  FOR M=1 TO N
330  J=J*2
340  IF J>=N THEN 370
350  NEXT M
370  FOR K=1 TO M
380  J2=2↑(K-1)
390  I2=N/(J2*2)
400  FOR I=0 TO I2-1
401  T(0)=F(0,Y)
410  FOR J=0 TO J2-1
500  X=N*I/I2+J
510  Y=X+J2
520  Z=J*I2
530  T(0)=W(0,Z)*F(0,Y)-W(1,Z)*F(1,Y)
540  T(1)=W(1,Z)*F(0,Y)+W(0,Z)*F(1,Y)
550  F(0,Y)=F(0,X)-T(0)
560  F(1,Y)=F(1,X)-T(1)
570  F(0,X)=F(0,X)+T(0)
580  F(1,X)=F(1,X)+T(1)
590  NEXT J
600  NEXT I
610  NEXT K
698  A=0
699  PRINT "HARMONIC          A(N)          B(N)"
700  FOR I=0 TO N/2-1
710  PRINT I,F(0,I)/32,F(1,I)/32
720  NEXT I
730  FOR I=0 TO N/2-1
736  V1=2*I/N
738  V2=F(0,I)/A
739  IF V2<1.000000E-03GO TO 760
740  PRINT I*2;V2
760  NEXT I
761  STOP
```

```
800  FOR I=0 TO N1
810  F(0,I)=0
820  NEXT I
900  DATA 0,32,16,48,8,40,24,56,4,36,20,52,12,44,28,60,2
901  DATA 34,18,50,10,42,26,58,6,38,22,54,14,46,30,62,1,33
902  DATA 17,49,9,41,25,57,5,37,21,53,13,45,29,61,3,35,19
903  DATA 51,11,43,27,59,7,39,23,55,15,47,31,63
998  RESTORE
999  GO TO 1
```

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